It should be pointed out that the poles of $H_{\min}^{(k)}(z)$ can be determined very accurately by first locating the corresponding poles in the w-plane and then transforming them to the z-plane [13].

REFERENCES

[1] L. R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975.

[2] F. Leeb, "Lattice wave digital filters with simultaneous conditions on amplitude and phase," in *Proc. 1991 IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, Toronto, Canada, May 1991, pp. 1645–1648.

[3] J. Földvári-Orosz, T. Henk, and E. Simonyi, "Simultaneous amplitude and phase approximation for lumped and sampled filters," *Int. J. Circuit Theory Applicat.*, vol. 19, pp. 77–100, 1991.

[4] B. Jaworski and T. Saramäki, "Linear phase IIR filters composed of two parallel allpass sections," in *Proc. 1994 IEEE Int. Symp. Circuits and Systems*, London, U.K., May 1994, pp. 537–540.

[5] A. Jones, S. Lawson, and T. Wicks, "Design of cascaded allpass structures with magnitude and delay constraints using simulated annealing and quasi-Newton methods," in *Proc. 1991 IEEE Int. Symp. Circuits and Systems*, Singapore, June 1991, pp. 2439–2442.

[6] S. S. Lawson and T. Wicks, "Design of efficient digital filters satisfying arbitrary loss and delay specifications," *Proc. Inst. Elect. Eng., Part G, Circuits, Devices, and Systems*, vol. 139, pp. 611–620, Oct. 1992.

[7] R. Nouta, "The Jaumann structure in wave-digital filters," *Int. J. Circuit Theory Applicat.*, vol. 2, pp. 163–174, June 1974.

[8] A. Fettweis, H. Levin, and A. Sedlmeyer, "Wave digital lattice filters," *Int. J. Circuit Theory Applicat.*, vol. 2, pp. 203–211, June 1974.

[9] L. Gazsi, "Explicit formulas for lattice wave digital filters," *IEEE Trans. Circuits Syst.*, vol. CAS-32, pp. 68–88, Jan. 1985.

[10] T. Saramäki, "On the design of digital filters as a sum of two all-pass filters," *IEEE Trans. Circuits Syst.*, vol. CAS-32, pp. 1191–1193, Nov. 1985.

[11] S. R. K. Dutta and M. Vidyasagar, "New algorithms for constrained minimax optimization," *Math. Programming*, vol. 13, pp. 140–155, 1977.

[12] K. Surma-aho, "Design of approximately linear phase recursive digital filters," M.Sc. Thesis, Dept. Elect. Eng., Tampere Univ. Technol., Finland, Apr. 1997.

[13] T. Saramäki, "Design of optimum recursive digital filters with zeros on the unit circle," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-31, pp. 450–458, Apr. 1983.

On the Design of the Target-Signal Filter in Adaptive Beamforming

Marcel Joho and George S. Moschytz

***Abstract*—This brief deals with the improvement of jammer suppression for adaptive broadband beamforming. The analysis is carried out with a two-microphone Griffiths–Jim Beamformer and a single jammer signal. By examining the derived optimal filter, a design strategy for the constant target-signal filter in the main channel of the adaptive beamformer is given. This leads to a substantial reduction in the required filter length of the adaptive filter, while maintaining the same jammer suppression. Furthermore, a faster rate of convergence can be achieved, as there are fewer filter coefficients that have to be adapted. The brief concludes with measurement results from a real-time implementation of an adaptive beamformer, showing that the proposed design method is effective, even when more than two microphones are used.**

***Index Terms*— Adaptive beamforming, array signal processing, Griffiths–Jim beamformer, microphone arrays, target-signal filter.**

I. INTRODUCTION

Adaptive broadband beamforming is becoming increasingly important in acoustical applications for spatial filtering [2]. Beamforming can be applied whenever multiple sound sources are present, which can be subdivided into two groups: target and jammer signals. The purpose of beamforming is to amplify a target signal while attenuating the jammer signals. Typical acoustical applications are in hearing aids and for videoconferencing, where one speaker is the target signal and the other speakers or noise sources located in the same room are considered jammers.

In [3], an adaptive beamformer is presented in which the filter coefficients are updated by an LMS algorithm in the time domain. After every adaptation step, the coefficient vector is adjusted such that the specified target signal transfer function in the frontal-direction is preserved. This additionally required operation after every coefficient update is elegantly circumvented in the Griffiths–Jim Beamformer [1] by the introduction of a *blocking matrix* for preprocessing the input signals. This leads to a simple update rule with no constraints on the adapted filter coefficients. Therefore any adaptive MISO-algorithm (multiple input single output) can be applied in a Griffiths-Jim beamformer [7].

Little attention has been paid to the influence of the constant *target-signal filter* $h_1(q)$ located in the main channel. Usually, a simple delay of l samples is chosen for this filter (i.e., $h_1(q) = q^{-l}$) to let the target signal pass through to the beamformer output without any distortion [1], [4].

This brief focuses on the role of the constant target-signal filter $h_1(q)$ in the Griffiths–Jim beamformer. It can be shown that the steady-state adaptation error strongly depends on the choice of $h_1(q)$. As a result, a design rule for the target-signal filter $h_1(q)$ is derived, which leads to shorter adaptive filters in the auxiliary channels and therefore to a faster adaptation toward the optimal filter coefficients.

Manuscript received October 16, 1998; revised March 9, 1999. This work was supported in part by the Swiss Federal Institute of Technology, ETH Zurich, Switzerland. This paper was previously presented at the IEEE International Symposium on Circuits and Systems, Monterey, CA, June 1998. This paper was recommended by Guest Editors F. Maloberti, P. Diniz, and K. Jenkins.

The authors are with the Signal and Information Processing Laboratory (ISI), Swiss Federal Institute of Technology, ETH Zurich, Switzerland.

Publisher Item Identifier S 1057-7130(99)05655-4.

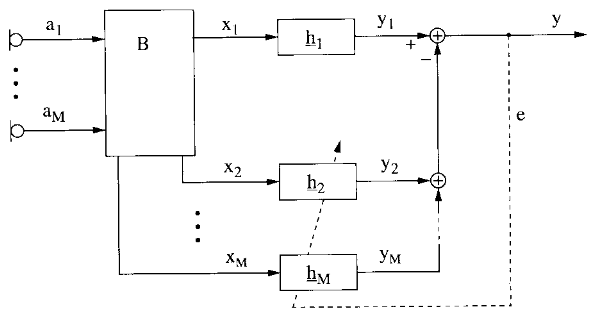

Fig. 1. Griffiths–Jim beamformer.

The drawback in this method is that the target signal is high-pass filtered. In an acoustical application, e.g., hearing aids, where improving speech intelligibility is the main objective, this is no real limitation, because low frequencies are of little importance. In Section II, the conventional Griffiths–Jim beamformer is described. In Section III, we analyze the two-microphone Griffiths–Jim beamformer and derive a design strategy for the target-signal filter. Measurements with a real-time implementation of an adaptive beamformer are presented in Section IV.

II. Adaptive Beamforming

A. Griffiths–Jim Beamformer

The system studied here is based on the adaptive beamformer described by Griffiths and Jim [1]. As seen in Fig. 1, the microphone array consists of M sensors. Its input signals a_m are transformed with a matrix B into a *main channel* x_1 and $M-1$ *auxiliary channels* $x_2 \cdots x_M$. B is an $M \times M$ matrix organized such that $x_m[t] = \Sigma_{i=1}^{M} b_{m,i} \cdot a_i[t]$ or, in vector notation, $\underline{x}[t] = B\underline{a}[t]$, where $\underline{a}[t] = (a_1[t], \cdots, a_M[t])^T$ and $\underline{x}[t] = (x_1[t], \cdots, x_M[t])^T$.

The matrix B is set up such that it prevents a *target* (in the form of a plane wave impinging perpendicularly on the array) from passing through to the auxiliary channels, while letting it pass unimpeded through to the main channel ($x_m[t] = 0$ for $m = 2, \cdots, M$ if $a_i[t] = a[t]$ for $i = 1, \cdots, M$). The matrix B without the first row is known as the *blocking matrix*. One possible realization of B is (as described in [1] for $M = 3$)

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}. \tag{1}$$

B. Time-Domain Filtering and Adaptation

Each channel x_m of the beamformer contains an finite impulse response (FIR)-filter $\underline{h}_m$ with N taps. The filter in the main channel $\underline{h}_1$ is assumed to be time invariant and designed to shape the target spectrum. $\underline{h}_2 \cdots \underline{h}_M$ are adaptive filters, updated after every time sample, such as to minimize the power of the beamformer output $y[t]$

$$y[t] = y_1[t] - \sum_{m=2}^{M} y_m[t] \tag{2}$$

$$y_m[t] = \underline{h}_m^T[t]\underline{x}_m[t] \tag{3}$$

$$\underline{x}_m[t] = (x_m[t], \cdots, x_m[t-N+1])^T \tag{4}$$

$$\underline{h}_m[t] = (h_{m,1}[t], \cdots, h_{m,N}[t])^T. \tag{5}$$

If perfect adaptation occurs, only the target signal filtered with $\underline{h}_1[t]$ remains at the beamformer output $y[t]$, whereas signal components from other spatial directions (*jammers*) vanish.

For the adaptation, it is assumed that target and jammers are uncorrelated; therefore, the beamformer output is taken as the adaptation error as well: $y[t] = e[t]$. The filter LMS update equations for real-valued input signals are

$$\underline{h}_m[t+1] = \underline{h}_m[t] + \mu_0 \underline{x}_m[t]e[t] \tag{6}$$

where μ_0 is the step-size controlling the rate of convergence and stability of the adaptation.

III. Two-Microphone Analysis

For the sake of simplicity, the Griffiths–Jim beamformer is analyzed with $M = 2$ microphones with one target and one jammer signal as shown in Fig. 2. As will be seen, the same conclusions hold for $M > 2$ microphones. Furthermore, it is assumed that the beamformer is located in an anechoic chamber, such that no reflections from the target and jammer signal appear at the microphones of the beamformer. The corresponding matrix B for $M = 2$ microphones is

$$B = \tfrac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{7}$$

The target signal is assumed to impinge perpendicularly on the array ($\theta = 0^\circ$) and, therefore, simultaneously at both microphones, whereas the jammer signal arrives with an intersensor delay τ at the two sensors, which is dependent on the angle of incidence θ

$$a_1(t) = s(t) + j(t) \tag{8}$$

$$a_2(t) = s(t) + j(t-\tau). \tag{9}$$

In a sampled-data system, the input signals are

$$a_1[t] = s[t] + j[t] \tag{10}$$

$$a_2[t] = s[t] + j[t-d] = s[t] + q^{-d}j[t] \tag{11}$$

where $d = \tau/T_s = \tau f_s$ is the intersensor delay normalized by the sampling period, T_s, q^{-1} is the delay-operator, and $[\cdot]$ denotes the sample index. Note that with this notation, d can be a rational number[1]. This means that the sequence of the jammer signal $j(\cdot)$ is sampled twice, both with the same sampling frequency but with a constant relative time-shift d depending on θ: $j(\cdot), j(\cdot - \tau)$ and $j[\cdot]$, $j[\cdot - d]$, respectively.

The beamformer output $y[t]$, which is also used as the adaptation error $e[t]$, is obtained from Fig. 2

$$y[t] = h_1(q)s[t] + \tfrac{1}{2}(h_1(q)(1+q^{-d}) - h_2(q)(1-q^{-d}))j[t]. \tag{12}$$

As can be seen from (12), the beamformer transfer function of the target signal is just $h_1(q)$, which is why it is named *target-signal filter*. As $h_1(q)$ is an arbitrary design parameter, a simple delay q^{-l} is normally chosen for it. The second term of (12) is the transfer function of the jammer signal. For its complete cancelation at the beamformer output, the transfer function must be equal to zero, which is true for

$$h_1(q)(1+q^{-d}) = h_2(q)(1-q^{-d}). \tag{13}$$

For a given $h_1(q)$, the optimal filter $h_2^o(q)$ and its z-transform becomes

$$h_2^o(q) = \frac{(1+q^{-d})}{(1-q^{-d})}h_1(q) \tag{14}$$

$$H_2^o(z) = \frac{(1+z^{-d})}{(1-z^{-d})}H_1(z). \tag{15}$$

The optimal filter $H_2^o(z)$ has a pole at $z = 1$ because of

$$1 - z^{-d}|_{z=1} = 0 \quad \forall d \tag{16}$$

[1] To prevent spatial aliasing, $-1 \le d \le 1$ must hold.

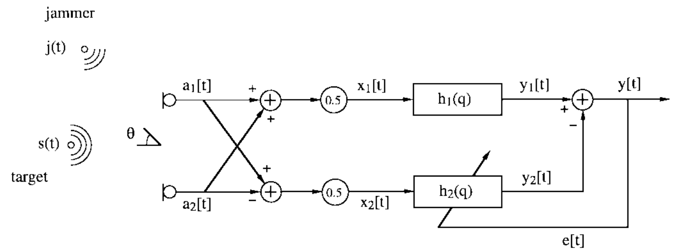

Fig. 2. Griffiths–Jim beamformer with two microphones.

and therefore has an infinite impulse response (IIR). If FIR filters are used, which is usually the case in adaptive beamforming, the optimum solution can only be approximated, and an adaptation error remains in the steady-state. To avoid this error from occuring, several methods have been proposed (e.g., [5], [6]).

One solution proposed in [5] is to insert a filter into the auxiliary channel $x_2[t]$ with a fixed or adaptive pole. The disadvantage of this method is that low-frequency sensor noise or a DC offset at the output of the microphone preamplifiers are strongly amplified in the auxiliary channels. Even in the case of a small adaptation error $H_2(z) - H_2^o(z)$, this type of signal appears as disturbing low-frequency noise at the output of the beamformer $y[t]$. As the beamformer output is also used for the adaptation of the filter coefficients, these are also affected by this noise, resulting in a low-frequency fluctuation.

The solution to the IIR-filter problem [see (15)] proposed in this brief is *to explicitly include a zero at unity in the target-signal filter, to prevent low-frequency signal components appearing at the output of the main channel* $y_1[t]$

$$H_1(z) = (1 - z^{-1})\overline{H}_1(z). \tag{17}$$

The filter $\overline{H}_1(z)$ remains a design parameter. It can still be chosen to specify the target transfer function $H_1(z)$ for higher frequencies and may be of linear phase. The optimal adaptive filter now results as

$$H_2^o(z) = \frac{(1 + z^{-d})}{(1 - z^{-d})}(1 - z^{-1})\overline{H}_1(z). \tag{18}$$

By substituting $z = e^{j2\pi f T_s}$ in $H_2^o(z)$ and evaluating the gain at $f = 0$, it is seen that this is now finite, in contrast to (15), for a general $H_1(z)$. It is difficult, in the general case, to show mathematically that

$$\|\tilde{H}_2(z)\| = \|H_2^{\#}(z) - H_2^o(z)\| \tag{19}$$

or

$$\|\tilde{h}_2(q)\| = \|h_2^{\#}(q) - h_2^o(q)\| \tag{20}$$

is smaller for $H_2^o(z)$ for (18) than for (15). The optimal FIR filter with N taps is denoted by $H_2^{\#}(z)$ and $h_2^{\#}(q)$ in the z-domain and time-domain, respectively. But what is obvious from (15) is that $H_2^{\#}(z)$ needs to approximate a pole at $z = 1$, which (18) does not. The fact that FIR filters can realize only sharp notches but not sharp peaks leads to the conclusion that the approximation error (19) or (20) is smaller with $H_2^o(z)$ from (18) than from (15).

In the special case when $d = 1$, which happens when the jammer impinges from *endfire* ($\theta = 90^\circ$) on the array, the optimal filter in (18) results in

$$H_2^o(z) = (1 + z^{-1})\overline{H}_1(z) \tag{21}$$

and is therefore FIR. This leads to the fact that $H_2^{\#}(z) = H_2^o(z)$ and therefore the jammer signal can be perfectly suppressed at the beamformer output. Note that this is never the case with (15) when (17) does not hold for $H_1(z)$, because in that case, $H_2^o(z)$ must be IIR.

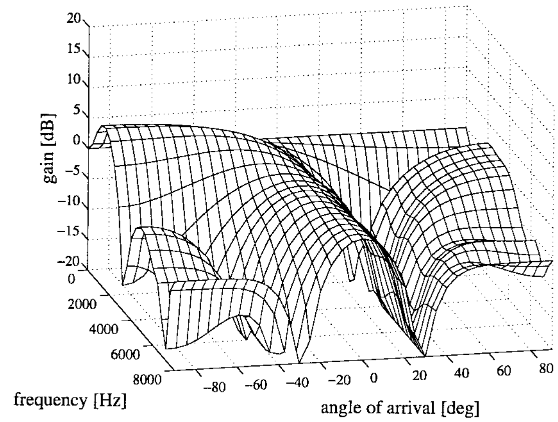

Fig. 3. Beampattern resulting after convergence using a delay as the target-signal filter. The jammer impinges onto the array from $\theta = 30^\circ$.

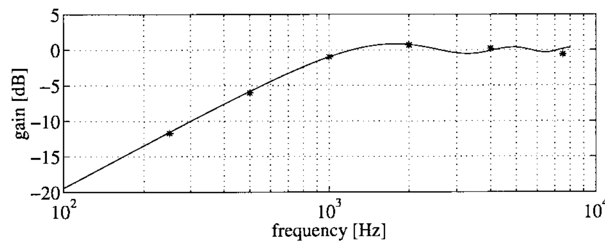

Fig. 4. Transfer function of the target-signal filter $H_1(z)$, which fulfills (17) with a zero at unity. The asterisks $*$ denote measured values of the transfer function.

Note that placing the first-order filter $(1 - z^{-1})$ after each microphone input $a_m[t]$, or after the beamformer output $y[t]$ is *not equivalent to including it in the target-signal filter* $H_1(z)$. It makes no difference for the target transfer function, but as seen from (12), it certainly does for the jammer transfer function.

IV. MEASUREMENTS

Simulation results with a two-microphone Griffiths–Jim beamformer showing the effectiveness of the proposed design method have already been given in [8]. In this section, results from real-world measurements are presented to verify the theoretical investigation of the target-signal filter $H_1(z)$. The adaptive beamformer of Fig. 1 with $M = 4$ microphones and $N = 10$ filter coefficients was implemented on a real-time DSP system. The distance between two adjacent microphones is $d_m = 2$ cm, which is just close enough to prevent *spatial aliasing* at the sampling frequency $f_s = 16$ kHz. The system is located inside an anechoic chamber. For the following measurements, a single jammer signal with band-limited $(f_s/2)$ white Gaussian noise impinging from $\theta = 30^\circ$ on to the microphone array is present. There is no target signal. The jammer signal source is a loudspeaker and the distance between the loudspeaker and the microphone array is 1.8 m. The power of the jammer signal is $\approx$ 80-dB SPL at each microphone. As required by the theory, the array was first calibrated such that each microphone has equal gain over a wide frequency range. To compare the influence of the target-signal filter, the system is examined in the steady-state. After convergence of the adaptive algorithm, the filter coefficients are used to compute the theoretical beampattern, which is the 2-D transfer function of the beamformer depending on the frequency f and angle of incidence θ of a sound source.

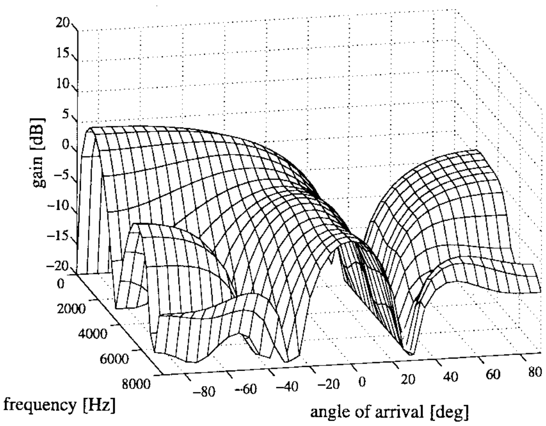

Fig. 5. Beampattern resulting after convergence using the target-signal filter from Fig. 4. The jammer impinges onto the array from $\theta = 30°$.

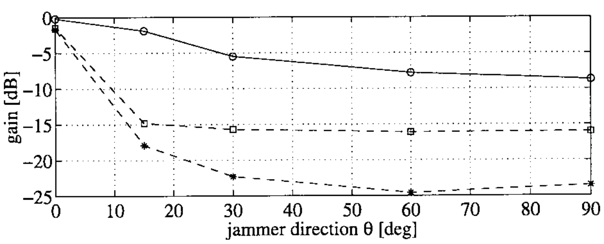

Fig. 6. Comparison of the achieved broadband jammer suppression after adaptation, depending on different jammer directions θ $(0°, 15°, 30°, 60°, 90°)$. o: initial broadband gain of the beamformer before adaptation. □: gain in jammer direction after adaptation if the target-signal filter is a simple delay. $*$: gain in jammer direction after adaptation with a target-signal filter transfer function, as in Fig. 4.

In the first case, a simple delay of $l = 4$ samples is chosen for the target-signal filter $h_1(q) = q^{-l}$. The resulting beampattern is shown in Fig. 3. The deep notch in the beampattern over a large frequency range at $\theta = 30°$ clearly shows the capability of jammer suppression at high frequencies. But as can also be seen, the beamformer is incapable of suppressing the low frequency part of the jammer signal. This is an inherent property of the beamformer, if only a simple delay is used for the target-signal filter.

In the second case, the chosen target-signal filter $H_1(z)$ has an explicit zero at $z = 1$, thereby fulfilling the constraint of the proposed filter design method (17). This forces the target-signal filter $H_1(f)$ to have zero gain at 0 Hz. The remaining design parameter $\overline{H}_1(z)$ is optimized such that $|H_1(f)|$ has about unity gain for higher frequencies. The transfer function of the target-signal filter $H_1(f)$ has high-pass characteristics and is shown in Fig. 4. The beampattern resulting after the adaptation is shown in Fig. 5. In contrast to Fig. 3, the low-frequency part of the jammer signal is now also suppressed. Of course, low frequencies of the target signal are also attenuated, but the corner frequency of the target-signal filter (see Fig. 4) can be pushed further down toward low frequencies by increasing the number of filter coefficients N.

In Fig. 6, the broadband jammer suppression achieved for different jammer directions θ are shown and compared to the initial gains of the beamformer. The initial gains are evaluated before the adaptation, when all adaptive filter coefficients are still zero. In the first case, where the target-signal filter is a simple delay, an improvement of the jammer suppression of 7–13 dB after adaptation is achieved, depending on θ. In the second case, where the target-signal filter of Fig. 4 is taken, the achieved jammer suppression could be further improved by up to 9 dB. The total achieved jammer suppression for $\theta > 20°$ is now 20 dB and more. Note that the deep notch in the beampattern at $\theta = 30°$ over the whole frequency range indicates that the array has been correctly calibrated. Although the analysis in Section III is carried out only with two microphones, the results in this section clearly show that the proposed filter-design method (17) can also be applied successfully to more than two microphones.

V. Summary

A design strategy for the target-signal filter in a Griffiths–Jim Beamformer is presented. It is shown that by a proper choice of this filter, namely high-pass characteristics with an explicit zero at unity, the pole of the optimal filter vanishes, resulting in a smoother transfer function. As adaptive FIR filters can only model steep notches but no sharp peaks, the task of adapting toward the optimal solution can thereby be simplified, and in special cases, perfect adaptation can even be achieved. The design strategy has been derived by analyzing a two-microphone beamformer, but it is valid also when multiple microphones are considered. Finally, measurement results of an adaptive beamformer, realized on a real-time DSP system, are given.

References

[1] L. J. Griffiths and C. W. Jim, “An alternative approach to linearly constrained adaptive beamforming,” *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 27–34, Jan. 1982.

[2] J. M. Kates, “A comparison of hearing-aid array-processing techniques,” *J. Acoust. Soc. Amer.*, vol. 99, pp. 3138-3148, 1996.

[3] O. L. Frost III, “An algorithm for linearly constraind adaptive array processing,” *Proc. IEEE*, vol. 60, pp. 926–935, Aug. 1972.

[4] J. E. Greenberg and P. M. Zurek, “ Evaluation of an adaptive beamforming method for hearing aids,” *J. Acoust. Soc. Amer.*, vol. 91, no. 3, pp. 1662–1676, Mar. 1992.

[5] R. P. Gooch, “Adaptive pole-zero array processing,” in *Proc. 16th Asilomar Conf. Circuits, Systems, and Computers*, Nov. 1982, pp. 45–49.

[6] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1985.

[7] M. Joho and G. S. Moschytz, “Adaptive beamforming with partitioned frequency-domain filters,” presented at the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics, New Paltz, NY, Oct. 19–22, 1997.

[8] ——, “On the design of the target-signal filter in adaptive beamforming,” in *Proc. IEEE Int. Symp. Circuits and Systems*, Monterey, CA, June 1998, vol. 5, pp. 166–169.