Additional notes to

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FDBD-I

This is an appendix to the publication [1]

In [1] it was described that the equations (13), (14), (11), and (15) can be embedded in the following equations:

$$\tilde{\mathbf{u}}_k = \tilde{\mathbf{W}}_k \tilde{\mathbf{x}}_k \tag{30}$$

$$\tilde{\mathbf{v}}_k = \tilde{\mathbf{W}}_k^H \tilde{\mathbf{u}}_k \tag{31}$$

$$\tilde{\mathbf{y}}_k = g(\mathbf{P}_\mathbf{y} \tilde{\mathbf{u}}_k) \tag{32}$$

$$\tilde{\mathbf{w}}_{k+1} = (1+\mu)\tilde{\mathbf{w}}_k - \frac{\mu}{L} \mathbf{P}_\mathbf{w} \, \tilde{\mathbf{V}}_k^H \tilde{\mathbf{y}}_k \tag{33}$$

where

$$\mathbf{P}_\mathbf{w} \triangleq \begin{bmatrix} \mathbf{I}_{L+1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_{3L-1} \end{bmatrix} \tag{28}$$

$$\mathbf{P}_\mathbf{y} \triangleq \begin{bmatrix} \mathbf{0}_{2L} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0}_L \end{bmatrix} . \tag{29}$$

For clarifi cation, we give the arrangement of the signal elements within the matrices in more detail to reveal the basice structure:

$$\left[\begin{array}{c} \dot{u}_{kL-4L+1} \\ * \\ \dot{u}_{kL-3L} \\ \hline u_{kL-3L+1} \\ * \\ u_{kL-2L} \\ u_{kL-2L+1} \\ * \\ u_{kL-L} \\ u_{kL-L+1} \\ * \\ u_{kL} \end{array}\right] = \left[\begin{array}{cccccccccccc} w_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_L & w_{L-1} & * \\ * & w_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_L & w_{L-1} \\ w_{L-1} & * & w_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_L \\ \hline w_L & w_{L-1} & * & w_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & w_L & w_{L-1} & * & w_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_L & w_{L-1} & * & w_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_L & w_{L-1} & * & w_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_L & w_{L-1} & * & w_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & w_L & w_{L-1} & * & w_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_L & w_{L-1} & * & w_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_L & w_{L-1} & * & w_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_L & w_{L-1} & * & w_0 \end{array}\right] \cdot \left[\begin{array}{c} x_{kL-4L+1} \\ * \\ x_{kL-3L} \\ x_{kL-3L+1} \\ * \\ x_{kL-2L} \\ x_{kL-2L+1} \\ * \\ x_{kL-L} \\ x_{kL-L+1} \\ * \\ x_{kL} \end{array}\right] \tag{30}$$

$$\left[\begin{array}{c} \dot{v}_{kL-3L+1} \\ * \\ \dot{v}_{kL-2L} \\ \hline v_{kL-2L+1} \\ * \\ v_{kL-L} \\ v_{kL-L+1} \\ * \\ v_t \\ \hline \dot{v}_{kL-4L+1} \\ * \\ \dot{v}_{kL-3L} \end{array}\right] = \left[\begin{array}{ccc|ccccccccc} w_0 & * & w_{L-1} & w_L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & w_0 & * & w_{L-1} & w_L & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_0 & * & w_{L-1} & w_L & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & w_0 & * & w_{L-1} & w_L & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_0 & * & w_{L-1} & w_L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & w_0 & * & w_{L-1} & w_L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_0 & * & w_{L-1} & w_L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_0 & * & w_{L-1} & w_L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_0 & * & w_{L-1} & w_L \\ \hline w_L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_0 & * & w_{L-1} \\ w_{L-1} & w_L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_0 & * \\ * & w_{L-1} & w_L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_0 \end{array}\right]^{*} \cdot \left[\begin{array}{c} \dot{u}_{kL-4L+1} \\ * \\ \dot{u}_{kL-3L} \\ \hline u_{kL-3L+1} \\ * \\ u_{kL-2L} \\ u_{kL-2L+1} \\ * \\ u_{kL-L} \\ u_{kL-L+1} \\ * \\ u_{kL} \end{array}\right] \tag{31}$$

$$\left[\begin{array}{c} 0 \\ * \\ 0 \\ 0 \\ * \\ 0 \\ \hline y_{kL-2L+1} \\ * \\ y_{kL-L} \\ \hline 0 \\ * \\ 0 \end{array}\right] = g\left(\mathbf{P_y}\left[\begin{array}{c} \dot{u}_{kL-4L+1} \\ * \\ \dot{u}_{kL-3L} \\ u_{kL-3L+1} \\ * \\ u_{kL-2L} \\ \hline u_{kL-2L+1} \\ * \\ u_{kL-L} \\ \hline u_{kL-L+1} \\ * \\ u_{kL} \end{array}\right]\right) \tag{32}$$

$$
\left[\begin{array}{c} w_0[k+1] \\ * \\ w_{L-1}[k+1] \\ w_L[k+1] \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right] = (1+\mu) \left[\begin{array}{c} w_0[k] \\ * \\ w_{L-1}[k] \\ w_L[k] \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right]
$$

$$
-\frac{\mu}{L}\mathbf{P_w} \left[\begin{array}{cccccc|ccc|ccc}
\dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} \\
\dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} & * \\
* & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} \\
\dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} & * & v_{kL} \\
\hline
v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} & * \\
* & v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} \\
v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} \\
v_{kL-L} & v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} & * \\
* & v_{kL-L} & v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} & v_{kL-2L+1} \\
v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * & \dot{v}_{kL-2L} \\
\dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1} & * \\
* & \dot{v}_{kL-2L} & v_{kL-2L+1} & * & v_{kL-L} & v_{kL-L+1} & * & v_{kL} & \dot{v}_{kL-4L+1} & * & \dot{v}_{kL-3L} & \dot{v}_{kL-3L+1}
\end{array}\right]^{*} \cdot \left[\begin{array}{c} 0 \\ * \\ 0 \\ 0 \\ * \\ 0 \\ \hline y_{kL-2L+1} \\ * \\ y_{kL-L} \\ \hline 0 \\ * \\ 0 \end{array}\right] \tag{33}
$$

Reference

[1] *M. Joho and P. Schniter, On frequency-domain implementations of filtered-gradient blind deconvolution algorithms, 36th Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, USA, Nov. 3–6, 2002, Vol. II, pp. 1653–1658.*

[2] *M. Joho and P. Schniter, Frequency-domain realization of a multichannel blind deconvolution algorithm based on the natural gradient, International Conference on Independent Component Analysis and Blind Signal Separation, ICA 2003, Nara, Japan, April 1–4, 2003, pp. 543–548.*